4.1) Expanding $(1 + x)^n$

Worked example

Find the binomial expansion of:

$(1 + x)^4$

$(1 - 2x)^5$

Your turn

Find the binomial expansion of:

$(1 + 3x)^6$

$1 + 18x + 135x^2 + 540x^3 + 1215x^4 + 1458x^5 + 729x^6$

Worked example

Find the first four terms in the binomial expansion of:

$$\frac{1}{1+x}$$

$$\frac{1}{(1-x)^3}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\frac{1}{(1+x)^2}$$

$$1-2x+3x^2-4x^3+\cdots$$

Worked example

Find the first four terms in the binomial expansion of:

$$\frac{1}{1+2x}$$

$$\frac{1}{(1-3x)^4}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\frac{1}{(1+4x)^3}$$

$1 - 12x + 96x^2 - 640x^3 + \cdots$

Worked example

Find the first four terms in the binomial expansion of:

$$\sqrt{1+x}$$

$$\sqrt[4]{1-x}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\sqrt[3]{1-x}$$

$$1-\frac{1}{3}x-\frac{1}{9}x^2-\frac{5}{81}x^3+\cdots$$

Worked example

Find the first four terms in the binomial expansion of:

$$\sqrt{1+6x}$$

$$\sqrt{1-4x}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\sqrt{1-2x}$$

$$1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \cdots$$

Worked example

State when the binomial expansion is valid:

$$\frac{1}{1+2x}$$

$$\frac{1}{(1-3x)^4}$$

$$(1+5x)^{\frac{3}{2}}$$

$$\frac{1}{\sqrt{1+\frac{x}{7}}}$$

Your turn

State when the binomial expansion is valid:

$$\frac{1}{(1+4x)^3}$$

$|x| < \frac{1}{4}$ or $-\frac{1}{4} < x < \frac{1}{4}$

$$\sqrt{1-\frac{x}{5}}$$

$|x| < 5$ or $-5 < x < 5$

Worked example

State when the binomial expansion is valid:

$$\frac{2-x}{\sqrt{1+3x}}$$

$$\frac{5+x}{(1-2x)^4}$$

$$(1+5x)^{\frac{3}{2}}\sqrt{1-\frac{x}{4}}$$

Your turn

State when the binomial expansion is valid:

$$\frac{2+x}{\sqrt{1+5x}}$$

$|x| < \frac{1}{5}$ or $-\frac{1}{5} < x < \frac{1}{5}$

$$(1-3x)^{\frac{5}{3}}\sqrt[4]{1+\frac{x}{2}}$$

$|x| < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$

Worked example

By substituting $x = 0.07$ into the binomial expansion for $\sqrt{1-4x}$, find a decimal approximation to $\sqrt{2}$

Your turn

By substituting $x = 0.01$ into the binomial expansion for $\sqrt{1-2x}$, find a decimal approximation to $\sqrt{2}$

$1.414213571\ldots$

Worked example

By substituting $x = 0.04$ into the binomial expansion for $\sqrt{1-4x}$, find a decimal approximation to $\sqrt{21}$ to 5 decimal places

Your turn

By substituting $x = 0.01$ into the binomial expansion for $\sqrt{1-8x}$, find a decimal approximation to $\sqrt{23}$ to 5 decimal places

4.79584

Worked example

Find the series expansion, in ascending powers of x, up to and including the x^3 term, of $\sqrt{1+7x}$

Find the percentage error in using $x = 0.01$ in this series expansion to estimate $\sqrt{107}$

Your turn

Find the series expansion, in ascending powers of x, up to and including the x^3 term, of $\sqrt{1+9x}$

Find the percentage error in using $x = 0.01$ in this series expansion to estimate $\sqrt{109}$

0.003% (3 dp)

Worked example

Find the x^2 term in the series expansion of:

$$\frac{5-x}{\sqrt{1-3x}}$$

Your turn

Find the x^2 term in the series expansion of:

$$\frac{2+x}{\sqrt{1+5x}}$$

$$\frac{65}{4}x^2$$

Worked example

Find the first three terms in the series expansion of:

$$\sqrt{\frac{1+2x}{1-3x}}$$

Your turn

Find the first three terms in the series expansion of:

$$\sqrt{\frac{1+x}{1-x}}, |x| < 1$$

$$1 + x + \frac{1}{2}x^2$$

Worked example

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{1-3x} - \frac{4}{1-2x}$$

Your turn

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{1+3x} - \frac{4}{1-5x}$$

$2 - 38x - 46x^2$

Worked example

In the expansion of $(1+kx)^{-3}$ the coefficient of x is 12. Find k

In the expansion of $(1+kx)^{-3}$ the coefficient of x^2 is 4 and $k>0$. Find k

Your turn

In the expansion of $(1+kx)^{-4}$ the coefficient of x is 20. Find k

$k=-5$